Appendices A and B

Accompanying "Karachi tides during the 1945 Makran tsunami"

CONTENTS

Introduction

Appendices A and B complement the body of this paper in describing detiding methods and detiding results more fully. Appendix A describes mathematically the detiding method that is outlined in the main text and diagrammed in Figure 4. Appendix B considers two additional detiding methods and their results. It is hoped that these details make it easy for others to replicate or adapt the approach that underpins the new results in the paper's Figure 5.

Definitions

Appendices A and B rely on a digitized tracing of supplementary Figure S1 — a scan of the Karachi marigram for November 15 to December 1, 1945. The time intervals between the digitized points range from less than one minute to three minutes. The complete set of times and heights from this digitization, listed in Table S1, is termed the *Digitized Marigram*.

The Digitized Marigram provides observed constraints on *extrema* — the times and heights of high waters and low waters. Sixty-five extrema, herein numbered 0 to 64, enclose the entire period of record of the marigram. Their predicted values are available from British Admiralty tide tables for the year 1945 (Hydrographic Department 1944, p. 106).

A tidal curve called the *Accepted Tide* is derived in Appendix A by combining observations and predictions of the 65 extrema. Points on the curve, evaluated at one-minute intervals, are listed in Table S2. The curve is a unique piecewise cubic polynominal through the extrema, which are accordingly called *control points* in the body of the paper and in Table S3. Values of the extrema in the Accepted Tide are listed in Table S3, along with underlying observed and predicted values for these same high waters and low waters.

Appendix B presents two additional tidal curves. The *Admiralty Tide*, like the Accepted Tide, is a unique piecewise cubic polynominal through the 65 extrema, but the extrema values in this case are taken directly from the Admiralty tide tables for Karachi. The Admiralty Tide is intended to approximate the tidal curve that might have been calculated from harmonic constituents, such as those compiled by the International Hydrographic Bureau (1966, sheet 123).

The *Pytides Tide*, described in B.4, is the result from using the harmonic constituents compiled by the International Hydrographic Bureau (1966, sheet 123) and the Pytides code, see Cox (2013), to produce a complete tide through the 65 extrema to see any differences from the *Admiralty Tide* that we used in our method to develop the Accepted Tide.

An *Alternative Tide* is computed to represent the common practice of detiding by means of a best-fit tidal curve through the observed data including the fluctuations ascribed to a tsunami. The steps used in this instance are described in Appendix B. Unlike the Alternative Tide, the Accepted Tide is completely independent of the tsunami.

Subtracting the Accepted Tide from the Digitized Marigram yields the *residual* plotted in Figure 5b. This residual is compared in Appendix B with two others, obtained with the Admiralty Tide and the Alternative Tide.

Most of the dates and times below are in India Standard Time (*IST*), 5.5 hours ahead of Coordinated Universal Time (UTC). The exceptions are where, for mathematical convenience, the dates and times of the 65 extrema have been converted into times, in hours, with respect to the great earthquake of 0327 hours on November 28, 1945 IST. Times *post-quake* are negative before the earthquake and positive thereafter.

Problems to be addressed

The detiding method described below is intended to mitigate several problems with the Digitized Marigram. The most basic of these problems is that the marigram is incomplete. Long gaps interrupt the marigram for November 20 (9 hours) and November 22-23 (14 hours). They include the time of extrema 21, 22, 31, and 32. Shorter gaps occur on November 28. The first of these, between extrema 50 and 51, spans five minutes near the onset of a gradual apparent drawdown, with respect to a rising tide, that begins before the great earthquake. The other two, soon after extremum 51, coincide with rapid fluctuations undoubtedly part of the 1945 Makran tsunami. Details about these various gaps are presented elsewhere (Atwater et al. 2018).

Another problem is that extrema in the Digitized Marigram commonly differ from extrema from the Admiralty tide tables. The differences seen outside the tsunami interval are large enough — as much as a few tens of minutes, and a few tenths of a meter — to produce visible effects on residuals during the tsunami interval.

The marigram for November 28 also includes two anomalies that could affect a best-curve fit through them. First is the apparent drawdown, with respect to a rising tide, between about 0145 and 0415 on November 28. Regardless of its cause, this puzzling anomaly should not be allowed to affect the tidal curve that is to be subtracted from the Digitized Marigram. Second, elevated water levels persist in the Digitized Marigram after the morning high tide of November 28 (extremum 51). As with the apparent drawdown, this persistent elevation must not influence the tidal curve to be used in the detiding.

Finally, an attempt to replicate the highs and lows of the Admiralty tide table for 1945 by using published harmonic constituents led to problems, as they required a shift in time of about an hour. This is described in detail in Section B.4 where we resolve the problem.

Appendix A Mathematical description of the detiding method

In what follows, the times and heights of the high and low tides needed for the method are described. Then in Sections A.1 and A.2 these times and heights are computed. Section A.3 describes how the method computes the tides between these highs and lows to complete the detiding and to form the residual.

The Admiralty tide tables for Karachi in 1945, based on tidal observations since the late 19th century, illustrate the harbour's mixed tides. The predictions for most days give two high tides of unequal height and two low tides of unequal height (main paper, Figure 3). To represent the predicted times and heights of the 65 extrema used in the detiding, we denote their times in hours post-quake as as t_0^A, t_1^A, $\ldots$, t_{64}^A and their heights in meters as h_0^A, h_1^A, $\ldots$, h_{64}^A. For instance, the last extremum (low water) before the quake was extremum 50. It is denoted as the point (t_{50}^A, h_{50}^A). Likewise, extremum 51 was the first higher high water post-quake with Admiralty prediction denoted by (t_{51}^A, h_{51}^A).

Using all 65 points $(t_j^A, h_j^A), j = 0 \ldots 64$, we can approximate the Admiralty Tide — defined above as the complete tidal curve through all these predicted extrema — by connecting each successive pair with a cubic polynominal. The resulting piecewise cubic would be the unique one that interpolated all these extrema and maintained zero first derivative at them. Such an Admiralty Tide could then be evaluated at all the times where we have marigram data and subtracted from this data to yield the residual. The residual would represent the non-tidal part of the marigram data.

Unfortunately, such a simple plan is flawed. The main reason, as shown in Appendix B, is that this Admiralty Tide does not match the marigram data well enough in the pre-tsunami interval (the interval around extrema numbers 2 to 50) or in the post-tsunami interval (the interval around extrema numbers 58 to 63) to accept the Admiralty Tide predicted extrema as close approximations to the real tide extrema in the tsunami interval (the interval around extrema numbers 51 to 57) or during the pre-tsunami gaps (extrema numbers 21, 22 and 31, 32). We also don't have data around extrema 0, 1 and 64, but these extrema are never used in our calculations so we have used the Admiralty Tide values for them in Tables S2 and S3.

The approach taken is to make an Accepted Tide that is still built from extrema 2 to 63, but agrees with fitted extrema from the Digitized Marigram outside the tsunami interval. The times and heights of the new accepted extrema are denoted (t_2, h_2) to (t_{63}, h_{63}) by deleting the A superscript used earlier to refer to the Admiralty Tide values. A unique piecewise cubic interpolating polynominal is then put through these points while maintaining a zero first derivative at them.

The extrema of the digitized marigram tracing outside the tsunami interval are ambiguous where the traced curve fluctuates at high water or low water. To reduce this ambiguity, a best least squares quadratic is fit to the marigram data in a 2 hour vicinity of each of the Admiralty Tide extrema 2 to 50 and 58 to 63, excluding the data gaps around 21, 22, 31, and 32. These 51 small least squares problems determine the extrema points (t_j, h_j), for $j = 2, 3 \ldots 20$, $j = 23, 24, \ldots 30$, $j = 33, 34, \ldots 50$, and for $j = 58, 59, \ldots 63$. These extrema are given in columns H, I, and J of Table S3.

The remaining task is to find (t_j, h_j) for $j = 21, 22, 31, 32$ in the pre-tsunami interval, and for $j = 51, 52, \ldots 57$ in the tsunami interval only using the information enumerated below.

1. $\{t_{17}^A, t_{21}^A, t_{25}^A\}$ for finding t_{21}
2. $\{t_{18}^A, t_{22}^A, t_{26}^A\}$ for finding t_{22}
3. $\{t_{27}^A, t_{31}^A, t_{35}^A\}$ for finding t_{31}
4. $\{t_{28}^A, t_{32}^A, t_{36}^A\}$ for finding t_{32}
5. $\{t_{47}^A, t_{51}^A, t_{59}^A\}$ for finding t_{51}
6. $\{t_{48}^A, t_{52}^A, t_{60}^A\}$ for finding t_{52}
7. $\{t_{49}^A, t_{53}^A, t_{61}^A\}$ for finding t_{53}

8. $\{t^A_{50}, t^A_{54}, t^A_{58}\}$ for finding t_{54}
9. $\{t^A_{47}, t^A_{55}, t^A_{59}\}$ for finding t_{55}
10. $\{t^A_{48}, t^A_{56}, t^A_{60}\}$ for finding t_{56}
11. $\{t^A_{49}, t^A_{57}, t^A_{61}\}$ for finding t_{57}
12. The 51 extrema points (t_j, h_j), for $j = 2, 3 \ldots 20$, $j = 23, 24, \ldots 30$, $j = 33, 34, \ldots 50$, and for $j = 58, 59, \ldots 63$ found from the least squares problems.

The 51 extrema points in item 12 above, like all the extrema in the Admiralty Tide or in the Accepted Tide, can be organized into the four "lunar-day" sets that are plotted in Figures 3 and 4 of the paper. Mathematically, they can be organized by dividing the extrema number by 4 and by using the remainder, m, as the set subscript. That is, if j modulo $4 = m$, extrema j is placed in set S_m. Sets S_0, S_1, S_2, and S_3 below correspond to the green, brown, blue, and purple sets of "lunar-day" points in Figures 3 and 4, respectively, that are known after the 51 least squares problems have been solved. These extrema are the fitted points that appear as the open circles in Figure 4a and their values can be found in columns H, I, and J of Table S3. The four sets are:

- $S_0 = \{4, 8, 12, 16, 20, 24, 28, 36, 40, 44, 48, 60\}$
- $S_1 = \{5, 9, 13, 17, 25, 29, 33, 37, 41, 45, 49, 61\}$
- $S_2 = \{2, 6, 10, 14, 18, 26, 30, 34, 38, 42, 46, 50, 58, 62\}$
- $S_3 = \{3, 7, 11, 15, 19, 23, 27, 35, 39, 43, 47, 59, 63\}$

A.1 Finding the times

The remaining times are found by making a simplifying assumption about ratios between successive time intervals between tides in each lunar-day set. Predicted intervals, and the ratios between them, are first obtained from the extrema times, t^A, in the Admiralty table. It is then assumed that these predicted ratios are equivalent to the ratios observed. This assumed equivalence, in turn, is used to derive times for the four extrema during long gaps in the Digitized Marigram (extrema 21, 22, 31, and 32) and for the seven extrema of the tsunami interval (51-57). For instance, to find t_{21}, note that 21 modulo 4 is 1, and set S_1 has the known values t_{17} and t_{25} that bracket t_{21}. Also, the Admiralty table times $\{t^A_{17}, t^A_{21}, t^A_{25}\}$ can be used. So, enforcing the ratio

$$r_{21} = \frac{t^A_{21} - t^A_{17}}{t^A_{25} - t^A_{17}} = \frac{t_{21} - t_{17}}{t_{25} - t_{17}} \tag{1}$$

gives t_{21} as

$$t_{21} = t_{17} + r_{21}(t_{25} - t_{17})\,. \tag{2}$$

For completeness, formulas for the remaining times are given below.

$$t_{22} = t_{18} + r_{22}(t_{26} - t_{18})\ ,\ r_{22} = \frac{t^A_{22} - t^A_{18}}{t^A_{26} - t^A_{18}} \tag{3}$$

$$t_{31} = t_{27} + r_{31}(t_{35} - t_{27})\ ,\ r_{31} = \frac{t^A_{31} - t^A_{27}}{t^A_{35} - t^A_{27}} \tag{4}$$

$$t_{32} = t_{28} + r_{32}(t_{36} - t_{28})\ ,\ r_{32} = \frac{t^A_{32} - t^A_{28}}{t^A_{36} - t^A_{28}} \tag{5}$$

$$t_{51} = t_{47} + r_{51}(t_{59} - t_{47}) \ , \ r_{51} = \frac{t^A_{51} - t^A_{47}}{t^A_{59} - t^A_{47}} \tag{6}$$

$$t_{52} = t_{48} + r_{52}(t_{60} - t_{48}) \ , \ r_{52} = \frac{t^A_{52} - t^A_{48}}{t^A_{60} - t^A_{48}} \tag{7}$$

$$t_{53} = t_{49} + r_{53}(t_{61} - t_{49}) \ , \ r_{53} = \frac{t^A_{53} - t^A_{49}}{t^A_{61} - t^A_{49}} \tag{8}$$

$$t_{54} = t_{50} + r_{54}(t_{58} - t_{50}) \ , \ r_{54} = \frac{t^A_{54} - t^A_{50}}{t^A_{58} - t^A_{50}} \tag{9}$$

$$t_{55} = t_{47} + r_{55}(t_{59} - t_{47}) \ , \ r_{55} = \frac{t^A_{55} - t^A_{47}}{t^A_{59} - t^A_{47}} \tag{10}$$

$$t_{56} = t_{48} + r_{56}(t_{60} - t_{48}) \ , \ r_{56} = \frac{t^A_{56} - t^A_{48}}{t^A_{60} - t^A_{48}} \tag{11}$$

$$t_{57} = t_{49} + r_{57}(t_{61} - t_{49}) \ , \ r_{57} = \frac{t^A_{57} - t^A_{49}}{t^A_{61} - t^A_{49}} \tag{12}$$

A.2 Finding the heights

The heights for extrema during gaps and during the tsunami interval are found by evaluating cubic splines described below for each of the four lunar-day sets at the times computed above. The set S_0 starts with the low water at extremum 4 and slowly varies to the lower low water at extremum 60. Set S_1 starts with the higher high water at extremum 5 and slowly varies to the high water at extremum 61. Set S_2 starts at the lower low water at extremum 2 and varies slowly to the low water at extremum 62. The set S_3 starts at the high water at extremum 3 and varies slowly to the higher high water at extremum 63. Since the heights within each set vary slowly, the points within each set can be interpolated with a slowly varying cubic spline. These splines are denoted $s_0(t)$, $s_1(t)$, $s_2(t)$, and $s_3(t)$ corresponding to the four sets S_0, S_1, S_2, and S_3, respectively, and t is the time in hours post-quake. They are depicted in the dotted green, brown, blue, and purple lines in Figure 4 to illustrate their use in computing the seven extrema heights h_{51} to h_{57} during the tsunami interval. Having just found the times for all the extrema, these splines can be evaluated at the appropriate times to compute the remaining heights. In particular, to find h_{21}, note that 21 modulo 4 is 1, so evaluating $s_1(t)$ at the time t_{21} gives h_{21}. Likewise, h_{22} is found by evaluating $s_2(t)$ at the time t_{22} since 22 modulo 4 is 2. That is,

$$h_{21} = s_1(t_{21}) \ , \ h_{22} = s_2(t_{22})\,. \tag{13}$$

The equations for the remaining heights are given below.

$$h_{31} = s_3(t_{31}) \ , \ h_{32} = s_0(t_{32}) \tag{14}$$

$$h_{51} = s_3(t_{51}) \ , \ h_{52} = s_0(t_{52}) \tag{15}$$

$$h_{53} = s_1(t_{53}) \ , \ h_{54} = s_2(t_{54}) \tag{16}$$

$$h_{55} = s_3(t_{55}) \ , \ h_{56} = s_0(t_{56}) \ , \ h_{57} = s_1(t_{57}) \tag{17}$$

This procedure is summarized graphically in Figure 4a and is illustrated more fully, for each of the four lunar-day sets, in Figure 4b. For instance, Figure 4b shows a blowup of $s_0(t)$ as the green dotted line and the green extrema 44, 48, and 60 from set S_0 along with the green computed extrema 52 and 56. Also shown is a blowup of $s_1(t)$ as the brown dotted line, the brown extrema 45, 49, and 61 from set S_1 along with the brown computed extrema 53 and 57. Likewise, shown is a blowup of $s_2(t)$ as the blue dotted line, the blue extrema 46, 50, 58, and 62 from set S_2 along with the blue computed extremum 54. Finally, a blowup of $s_3(t)$ is shown as the purple dotted line, the purple extrema 43, 47, 59, and 63 from set S_3 along with the purple computed extrema 51 and 55.

A.3 Completing the Accepted Tide

All the extrema have now been computed, and as mentioned earlier, the tide is completed by putting a unique piecewise cubic polynominal through these extrema, guaranteeing the derivative of this piecewise cubic is zero at them. This accepted tide can then be evaluated at any time $t_2 \leq t \leq t_{63}$. If we evaluate it at the marigram times and subtract these times from the marigram data we get the residual. This accepted tide and residual at the times of the marigram data can be found in the last two columns of Table S1.

This piecewise cubic polynominal can be continued backward in time by adding the unique cubic between least-squares extrema 2 and Admiralty Tide extrema 1 (with zero derivatives at the extrema) and the unique cubic between Admiralty Tide extrema 1 and Admiralty Tide extrema 0. Likewise, it can be continued forward in time by adding the unique cubic between least-squares extrema 63 and Admiralty Tide extrema 64. The resulting Accepted Tide, evaluated at intervals of 1 minute from $t_0 \leq t \leq t_{64}$, is given in the last column of Table S2 and is graphed in light blue in Figure 5a.

Appendix B Comparisons among detiding results

As noted in the body of the paper, the new residual plotted in Figure 5 differs from two prior detidings of the Karachi marigram of the 1945 Makran tsunami. In those two prior results, from Neetu et al. (2011) and Heidarzadeh and Satake (2015), the detided tsunami spans a nine-hour interval on November 28 IST. This interval begins a little more than one hour after the great earthquake, the first wave is a positive one, and the main waves oscillate more or less symmetrically above and below the datum of the residual. A very different residual is obtained in Figure 5 on the basis of the Accepted Tide that is derived in Appendix A and Figure 4. The residual covers most of November 15 to December 1, begins with a negative wave — an apparent drawdown before the earthquake — , and the initial waves oscillate asymmetrically.

This appendix examines the reproducibility of the apparent drawdown and the relative amplitudes of the initial waves in the residual derived with the Accepted Tide. The appendix compares the Accepted Tide residual with those derived from two additional tidal reconstructions introduced above; namely, the Admiralty Tide and and Alternative Tide.

The comparisons are graphed in this appendix at two time scales: an overview from November 15 to December 1 (Figure B1; extrema 0-64), and a focus on the 24 hours before and after the great earthquake of November 28 (Figure B2; extrema 47-54). Individual points of the Digitized Marigram, and of the residuals derived from it, become visible where the scanned marigram was digitized at time intervals of 2-3 minutes, and where the graphed water levels vary rapidly.

In Section B.1, the Alternative Tide is derived. Section B.2 compares the Admiralty Tide, the Accepted Tide, and the Alternative Tide to the Digitized Marigram outside the tsunami interval. These three tides are further compared in their effects on the apparent drawdown and on height estimates for positive waves in Section B.3. In Section B.4, we verify the highs and lows and shape of the Admiralty Tide we used by evaluating known harmonic constituents. In doing so, we point out how one must be careful in using these published constituents since around a one hour shift to the right in the resulting curve is necessary. The results of Section B.4 are illustrated in Figures B3, B4, and B5 for the same extrema as shown in Figure B1 and in Figure B6 for extrema 43-64 which all occur between 48 hours before and 90 hours after the great earthquake of November 28.

B.1 Derivation of an Alternative Tide

The Alternative Tide is derived in four steps. First, an initial residual is obtained by subtracting the predicted tide (in our case, the Admiralty Tide) from the Digitized Marigram. Second, a slowly varying piecewise polynominal of degree 15 is fit to this initial residual. Third, the slowly varying polynominal is subtracted from the initial residual and added to the Admiralty Tide. The resulting tide provides a new set of values for extrema 2 through 63. In the final step, these extrema are connected with a piecewise cubic polynominal in the same fashion described in Appendix A. This final result is plotted in magenta in Figures B1 and B2. The values of its extrema are listed in columns Q, R, and S of Table S3.

B.2 Extrema comparisons and Residual comparisons

Comparing the tidal extrema to the Digitized Marigram extrema found by the quadratic fits outside the tsunami interval and the pre-tsunami gaps at 21, 22, 31, and 32 measures how closely the Admiralty Tide and the Alternative Tide agree with the Accepted Tide in this region. Recall, the Accepted Tide adopted these same Digitized Marigram extrema so it has perfect agreement.

Extrema comparisons

At both time scales plotted in Figures B1 and B2, the Admiralty Tide (green) visibly departs from the Digitized Marigram (gold) and the Accepted Tide (blue) at many of the extrema outside the tsunami interval. The mismatch becomes more apparent as the time scale and height scale expand in Figure B2. On the basis of data in columns D (predicted) and J (observed) in Table S3, the largest height differences outside the tsunami interval are with extrema 6 (21 cm) and 43 (18 cm). These amount to large fractions of the maximum peaks and troughs obtained by detiding (Figure 5). Time differences can be computed using columns B (predicted) and H (observed) in Table S3. Significant differences for the Admiralty Tide are 27, 22, 14, 15, 20, and 16 minutes at extrema 25, 41, 47, 50, 60, and 61, respectively.

Columns S (Alternative) and J (observed) can be used to find the extrema height differences for the Alternative Tide extrema. These differences are significantly smaller than for the Admiralty Tide. Some of the largest height differences are with extrema 5 (9 cm), 9 (9 cm), 42 (5 cm), and 59 (5 cm). Time differences can be computed using columns H (observed) and Q (Alternative) in Table S3. Significant differences for the Alternative Tide are 24, 16, 19, 18, 21, 16, 22, 20 and 15 minutes at extrema 25, 30 37, 39, 41, 42, 46, 60, and 61, respectively, but only 6 and 7 minutes at extrema 47 and 49 closer to the earthquake. The Alternative Tide is plotted in magenta in Figures B1 and B2.

Residual comparisons

Comparing the Admiralty, Accepted, and Alternative residuals outside the tsunami interval and the pre-tsunami gaps at 21, 22, 31, and 32 measures how closely the respective tides compare with the Digitized Marigram data in Column C of Table S1 at the *same* time. If the Marigram data just measured the tide, the residual would be zero for a perfectly calculated tide. Of course, this will not be completely true for various reasons, including noise in the Marigram caused by various sources as well as an imperfectly calculated tide. However, in the region where we do this comparison, there is no effect due to the tsunami pre-quake and little effect from the tsunami post-quake from extrema 58-63. Also, the region we consider will stop 5 hours pre-quake (to avoid the drawdown region) and not begin again until after extrema 58 on December 30.

From Column E of Table S1, we find the Accepted Residual to have a maximum value of 9 cm in the pre-quake interval [-48, -24] hrs, 9 cm in the pre-quake interval [-24, -5] hrs, and 9 cm in the post-quake interval [46.6, 80] hrs. Supplementary Table S1 does not give the Admiralty nor the Alternative residuals, but they are plotted in Figures B1 and B2. The corresponding values for the Admiralty residual are 26 cm in the pre-quake interval [-48, -24] hrs, 19 cm in the pre-quake interval [-24, -5] hrs, and 19 cm in the post-quake interval [46.6, 80] hrs. Finally, the corresponding values for the Alternative residual are 13 cm in the pre-quake interval [-48, -24] hrs, 8 cm in the pre-quake interval [-24, -5] hrs, and 16 cm in the post-quake interval [46.6, 80] hrs.

In the region considered above, the Admiralty Tide does not match the Digitized Marigram data at the extrema or at times between them as well as either the Accepted or Alternative Tides. Furthermore, the Accepted Tide matches better at the extrema of the Digitized Marigram by design. The discussion below compares these Tides in the regions of the drawdown and early tsunami waves.

B.3 Effects on the apparent drawdown and ensuing positive waves

Overall, outside the tsunami interval, the Accepted Tide agrees more with the Digitized Marigram than do the Admiralty Tide and the Alternative Tide as seen from the residuals in Figure B1c. This agreement is particularly important in the hours before the apparent drawdown of 0145-0415 on November 28 (Figure B2c). The Accepted Tide obtains this agreement by using the observed low water at the midnight between November 27 and 28 (extremum 50), and by intentional neglect of the aparent drawdown in its calculation.

The Admiralty Tide differs from the Digitized Marigram before the apparent drawdown because, for the low water of extremum 50, the prediction is 8 cm higher than the observation (Table S3, cell D53 compared to cell J53).

Curve fitting to non-tidal anomalies compromises the Alternative Tide early on November 28. First, by passing through the apparent drawdown and the ensuing positive wave, it produces an underestimate of the amount of drawdown, as well as an overestimate of the height of the first positive tsunami wave. The Alternative Tide also attempts to fit the mainly positive tsunami waves that followed the high tide of extremum 51. This attempt produces a lag, of nearly 30 minutes, in the time of extremum 51 in the Alternative Tide.

The Alternative Tide and the Accepted Tide converge later in the morning of November 28 (Figure B2b). They nearly agree, for instance, at 0815, the time of the maximum water levels reported in newspapers. A residual for these water levels cannot be determined from the Digitized Marigram because the gauge was out of order at the time, reportedly because of a mechanical problem caused by the tsunami (Atwater et al. 2018). But whatever the water level at 0815, residuals derived from all three tidal curves would give roughly the same tsunami height for it. Evaluated at 0815, the Accepted Tide, Alternative Tide, and Admiralty Tide values are 2.16, 2.18, and 2.23 meters, respectively. Residuals computed with these values would thus differ by 7 centimeters at most.

B.4 Use of harmonic constituents

The method we used to develop the Accepted Tide that agreed with the gauged water levels outside the tsunami window of November 28 and 29 used the predicted highs and lows of the 1945 Admiralty tide table. We connected these highs and lows with piecewise cubics to produce an Admiralty Tide at all points between these extrema. Our method depended on the time spacing between these published highs and lows, but did not depend on their heights. It is reasonable to ask why we did not use known tidal constituents to construct this predicted Admiralty Tide (thereby verifying the published times of the highs and lows), and whether the shape of the resulting curve would be different from that produced by the piecewise cubics. Below, we describe the results of using the published constituents to actually do this, and give the differences from the resulting curve to the Admiralty Tide we used. The results verify that the Admiralty Tide extrema times we used are unlikely to be off by more than 24 minutes, and the resulting shapes of the curves are similar. We also describe how to use these published constituents within a modern computer program to evaluate the tide, as we believe this would be important to anyone trying to repeat this experiment.

To be clear, we are not concerned with trying to find values of constituents by fitting the gauged water levels of the marigram to get an Accepted Tide, because we believe we do not have enough marigram data to do so. Instead, we are only interested to know if the published constituents can be used to reproduce the Admiralty Tide that we used in our own method for finding the Accepted Tide.

The latest tidal constituents available publicly for Karachi were the 35 constituents compiled by the International Hydrographic Bureau (1966:sheet 123) (IHB). In principle, when computing its 1945 tide tables for Karachi (Hydrographic Department 1944:293), the Admiralty probably used these same constituent names and values, because both sources state that the underlying observations were made between 1868 and 1920. In that case the Admiralty tables neglect effects of modifications to Karachi Harbour between 1920 and 1945.

We used the Pytides code of Cox (2013) to evaluate the Karachi tidal curve using the 35 IHB constituents with the published IHB information that the phases were related to the UTC+5:30 time. The resulting curve, when shifted roughly an hour to the right, reproduced amplitudes and times of the Admiralty table highs and lows with mean differences of 0.19 (ft) and 6.6 (min), respectively, as described below. Furthermore, the publications by Eccles (1901a, pp. 294) and Eccles (1901b, pp.10-18) helped resolve this necessary shift.

Secular changes in tidal constituents at Karachi were considered by Eccles (1901b, pp. 10-18). He updated the values of these same 35 constituents every two years between 1868 and 1892. These values change slightly with each update, and they differ somewhat from those updated to 1920 in the 1966 IHB compilation. For example, the Eccles value for the amplitude of M2 (the constituent with the highest amplitude) in the 1885-1886 constituent rendering was 2.552 and the 1966 IHB value was 2.566. The Eccles value for the phase of M2 in 1885-1886 was 292.5 and the 1966 IHB value was 294.0. The value of the mean tidal level Eccles used in 1885-1886 was 7.21 (5.21 for our datum) and this was the same as the 1966 IHB value. Since the speed of the M2 constituent is slightly more than 28.98 degrees/hr, the M2 phase difference between Eccles and IHB is not enough to account for the needed shift of around 1 hour described above. Similar statements can be made for the other 1885-1886 constituents.

In 1885-1886, the IST time zone (UTC+5:30) had not yet been established, and the local time was measured in (UTC+4:28:12). Therefore, the published 1885-1886 constituents which are similar to those published in 1966 could not have been related to UTC+5:30. Eccles (1901a, p. 294) outlines how the calculations of the constituent argument was done. The longitude of the observations was used. Eccles (1901b, p. 9) gives the longitude of the Karachi tide gauge as 66°, 58′ East. This corresponds to the time the phases are related to as (UTC+4:27:52) which is only 20 seconds from the way the local time then was measured.

We recomputed the predicted tide with the IHB values assuming the phases were related to the longitude of 1945 observations (66.97° E) which is UTC+4:27:52, not to the 82.5° E meridan for the time zone used in Karachi in 1945 (India Standard Time IST=UTC+5:30). This means 1 hour, 2 minutes, and 8 seconds should be subtracted from the IST times of interest, and Pytides called using these UTC+4:27:52 times. The resulting curve will be in UTC+4:27:52 time, and if we are interested in plotting the results in hours post quake, the time of the quake (03:37 IST November 28) is taken as UTC+2:24:58 on November 28. Note that this does not entail changing the amplitudes or the phases of the published constituents that are given to Pytides. (We also note that the same results can be achieved by sending the IST times of interest to Pytides directly, and then shifting the resulting curve to the right by 1 hour, 2 minutes, and 8 seconds. This also does not entail changing the amplitudes or the phases of the published constituents before Pytides use.)

Four accompanying graphs show the Pytides results. The first three compare extrema only, Pytide Tide vs Admiralty tide tables, for 65 extrema between November 15 and December 1 (Figures B3-B5). The fourth graph focuses on the shapes of complete tidal curves from 48 hours before to 90 hours after the great 1945 Makran earthquake of 0327 IST on November 28 (Figure B6). In this plot, the Admiralty extrema have been connected with piecewise cubics, to see whether the resulting Admiralty Tide resembles the Pytides Tide.

These Pytides results reveal no major problems with the Admiralty tide table predictions, nor with the piecewise cubic shapes. In the comparisions of the 65 extrema (Figures B3-B5), the mean height difference, Pytides vs. Admiralty, is 5.9 cm (0.19 ft), and the mean time difference is 6.6 minutes. The maximum differences are 17.4 cm (.57 ft) and 23.7 minutes. The differences are greatest with the high and higher high waters of lunar-day sets S_1 and S_3, and they are least with the lower low and low waters of lunar-day sets S_2 and S_4. These lunar-day sets are defined in Appendix A, and are identified also in Figures 3 and 4 of the paper. Extrema 50 and 51 that are important in bracketing the drawdown of water on November 28, are in lunar-day sets S_2 and S_3 and had differences of 16.4 and 9.9 minutes, respectively, with the Pytides extrema occuring later. As for tidal-curve shapes, the comparison in Figure B6 shows them closely approximated by piecewise cubics. That is, piecewise cubics produce an Admiralty Tide shaped much like the Pytides Tide that is based on constituents. This last observation is important, as piecewise cubics are also used to complete our Accepted Tide once its extrema are calculated.

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Figure captions

Figure B1

Comparison of three approaches to detiding the entire Karachi marigram of Figure S1

Figure B2

As in Figure B1, but focused on the 24 hours before and after the great Markan earthquake of 0327 hours, November 28, 1945, India Standard Time

Figure B3

Pytides Tide using the International Hydrographic Bureau constituents (1966) from 320 hours pre-quake to 90 hours post-quake. The green circles are the Hydrographic Department (1944) published Admiralty table highs and lows.

Figure B4

The 1944 Admiralty table minus the Pytides extrema heights. Pytides used the International Hydrographic Bureau (1966) constituents for Karachi. The difference is in feet, and the horizontal axis is the extremum number 0 to 64.

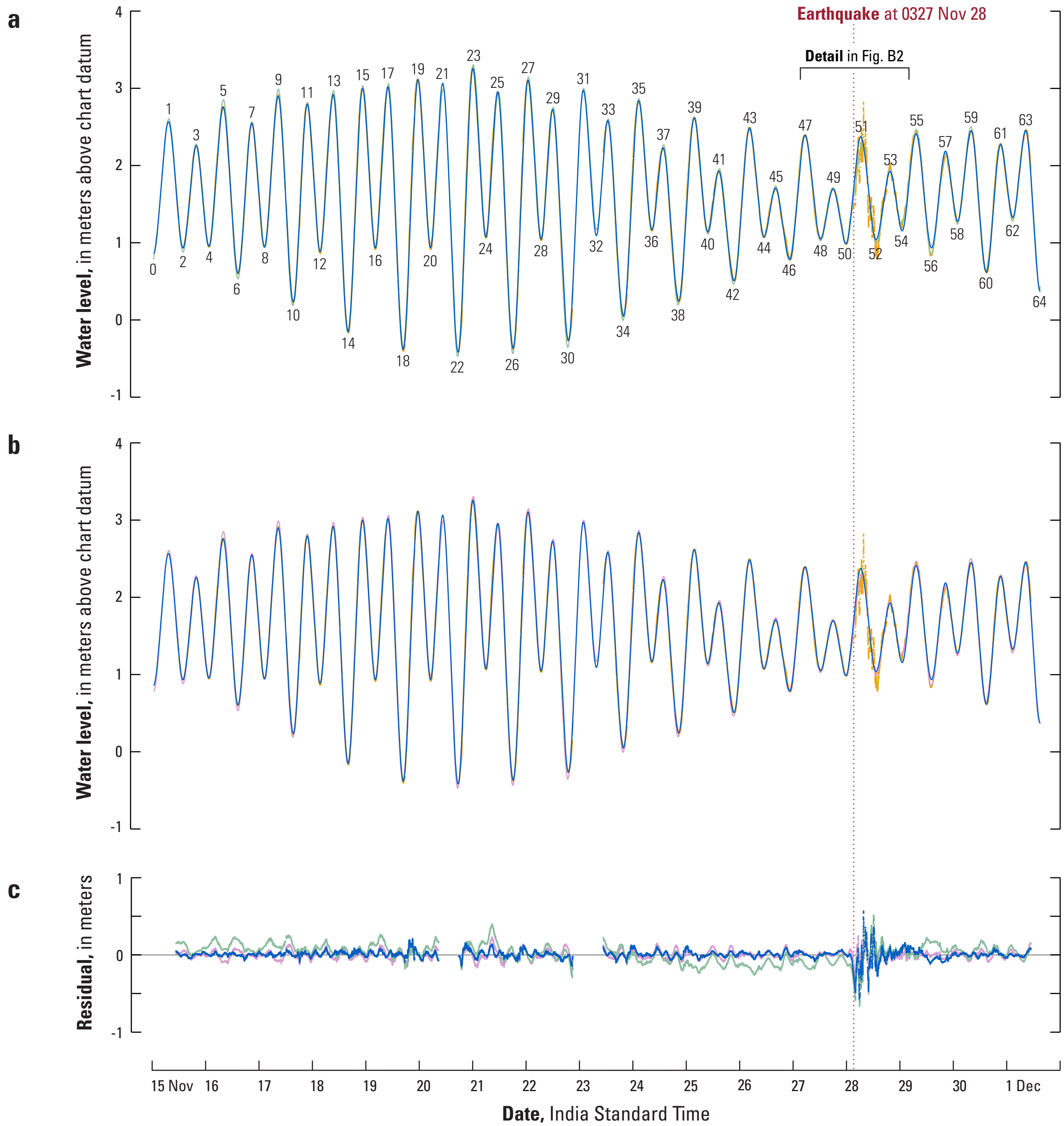
Figure B5

The 1944 Admiralty table minus the Pytides extrema times. Pytides used the International Hydrographic Bureau (1966) constituents for Karachi. The difference is in minutes, and the horizontal axis is the extremum number 0 to 64.

Figure B6

The Admiralty Tide created by connecting the 1944 Admiralty highs and lows with a piecewise cubic and the Pytides Tide created using the International Hydrographic Bureau (1966) constituents for Karachi from 48 hours pre-quake to 90 hours post-quake. The mean tide level was 5.21 feet.

Figure B1



EXPLANATION

Computed tidal curve—Defined more fully in text

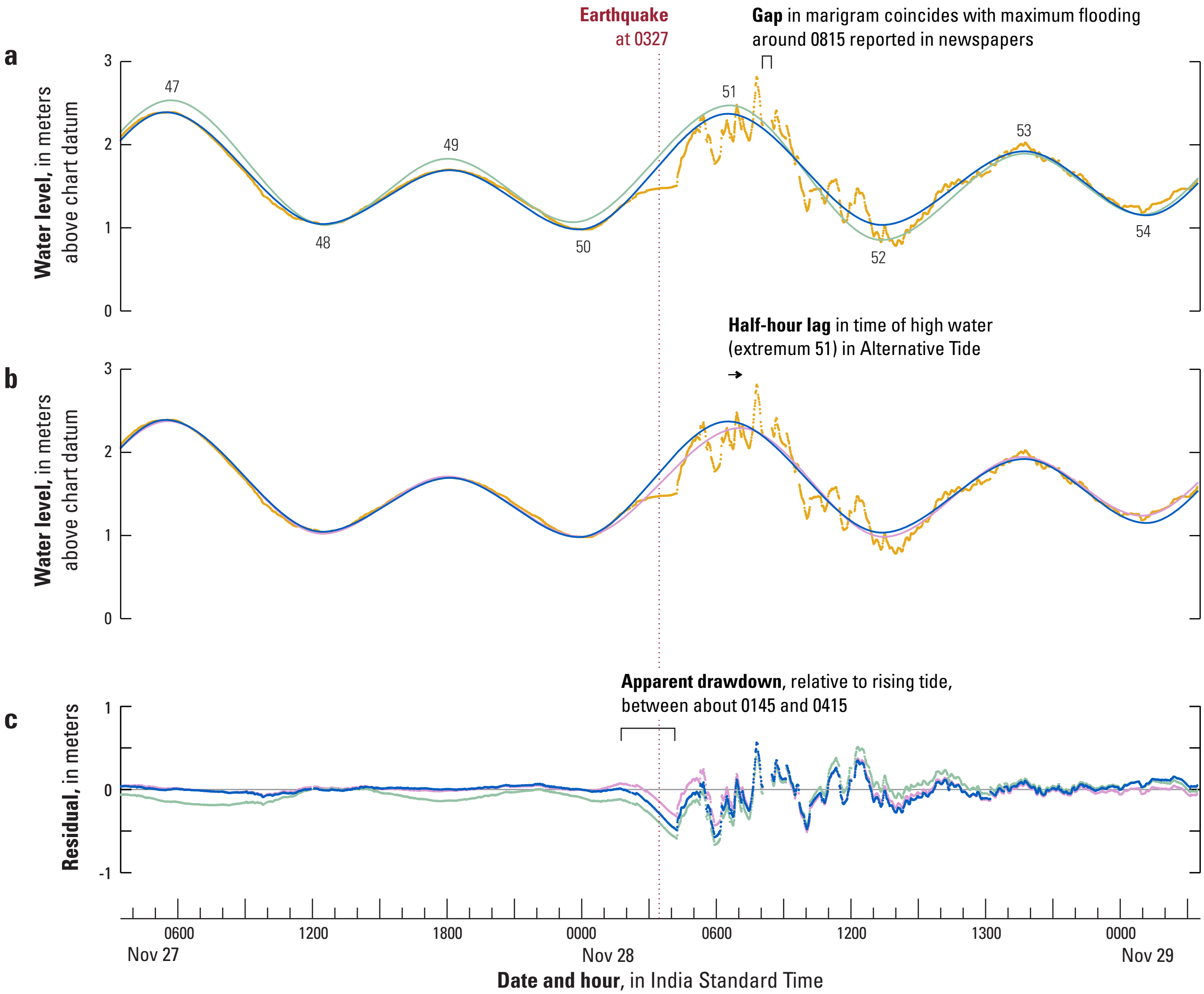
- **Accepted Tide**—Controlled by tide tables and by extrema before and after tsunami
- **Admiralty Tide**—Controlled by predictions in Admiralty tide tables. Plotted in **a**
- **Alternative Tide**—Incorporates curve fitting to Digitized Marigram. Plotted in **b**

Observed tidal data (Digitized Marigram)—Points digitized at intervals of one to three minutes along tracing of scanned marigram (scan in Fig. S1)

22 **Extremum**—High water (odd) or low water (even) numbered as in Table S3

Residual—Difference between computed tidal curve and Digitized Marigram. Presented as series of points. Color keyed to computed tidal curve

Figure B2



EXPLANATION

Computed tidal curve—Defined more fully in text

Accepted Tide—Controlled by tide tables and by extrema before and after tsunami

Admiralty Tide—Controlled by predictions in Admiralty tide tables. Plotted in **a**

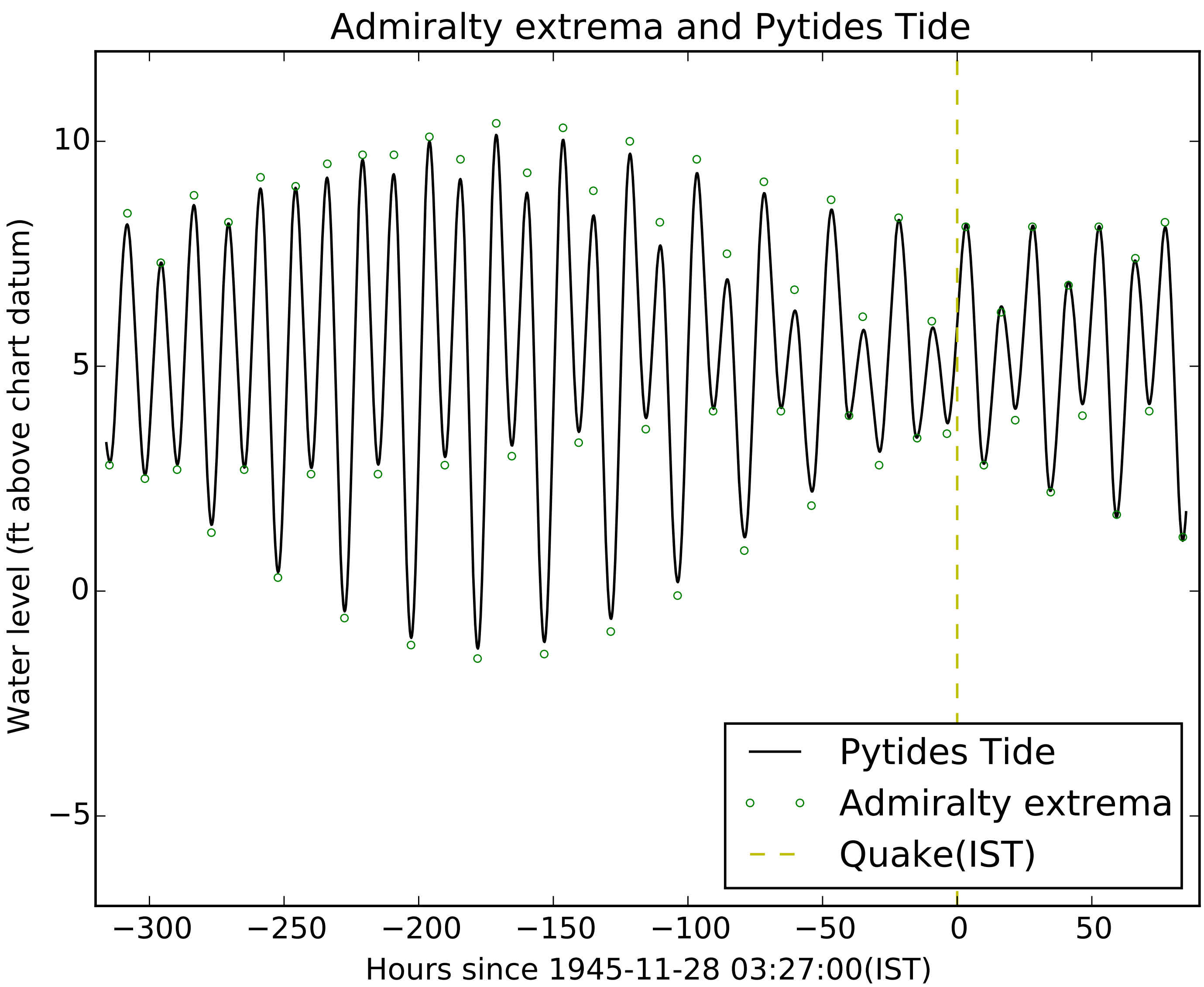
Alternative Tide—Incorporates curve fitting to Digitized Marigram. Plotted in **b**

Observed tidal data (Digitized Marigram)—Points digitized at intervals of one to three minutes along tracing of scanned marigram (scan in Fig. S1)

50 **Extremum**—High water (odd) or low water (even) numbered as in Table S3

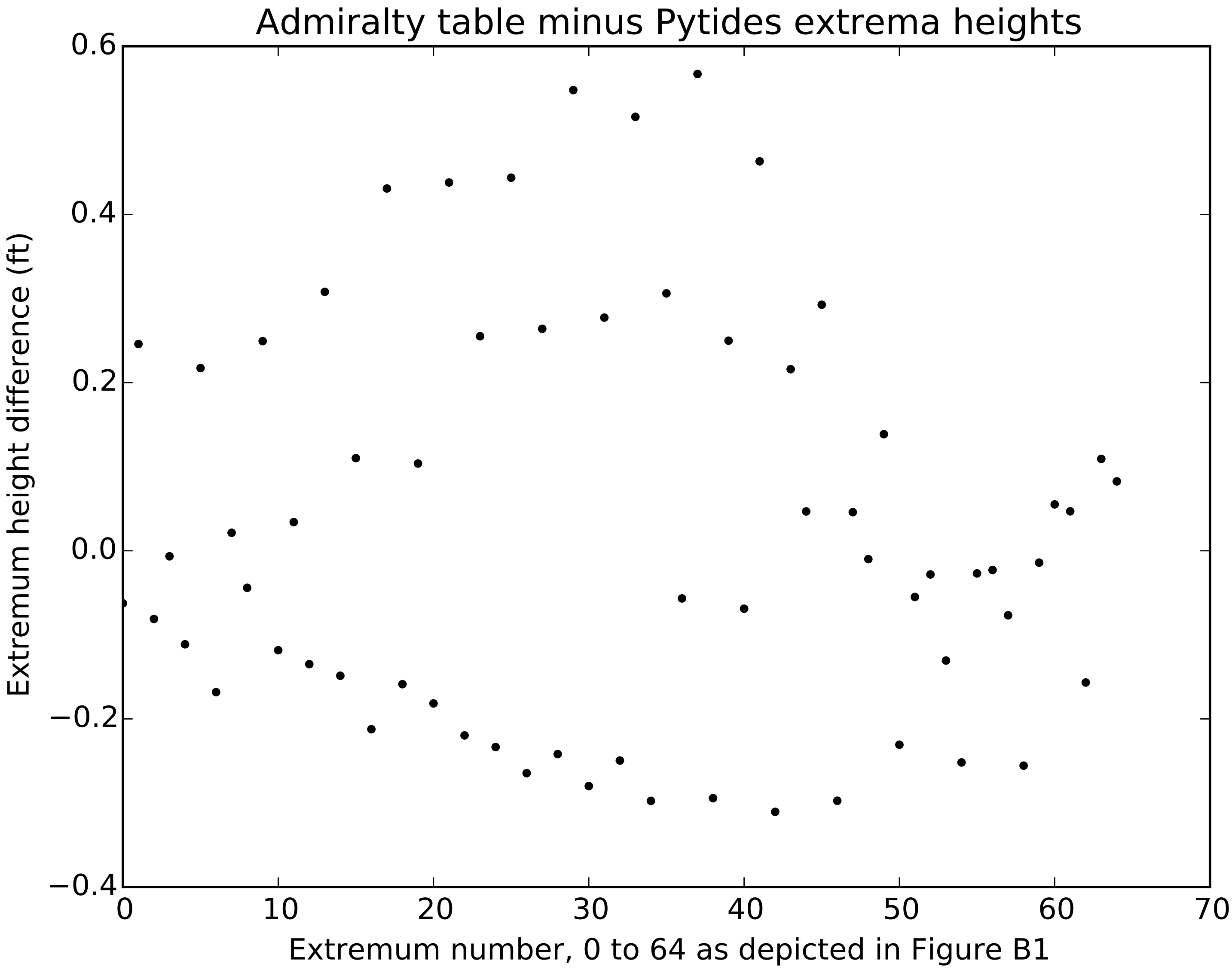
Residual—Difference between computed tidal curve and Digitized Marigram. Presented as series of points. Color keyed to computed tidal curve

Figure B3



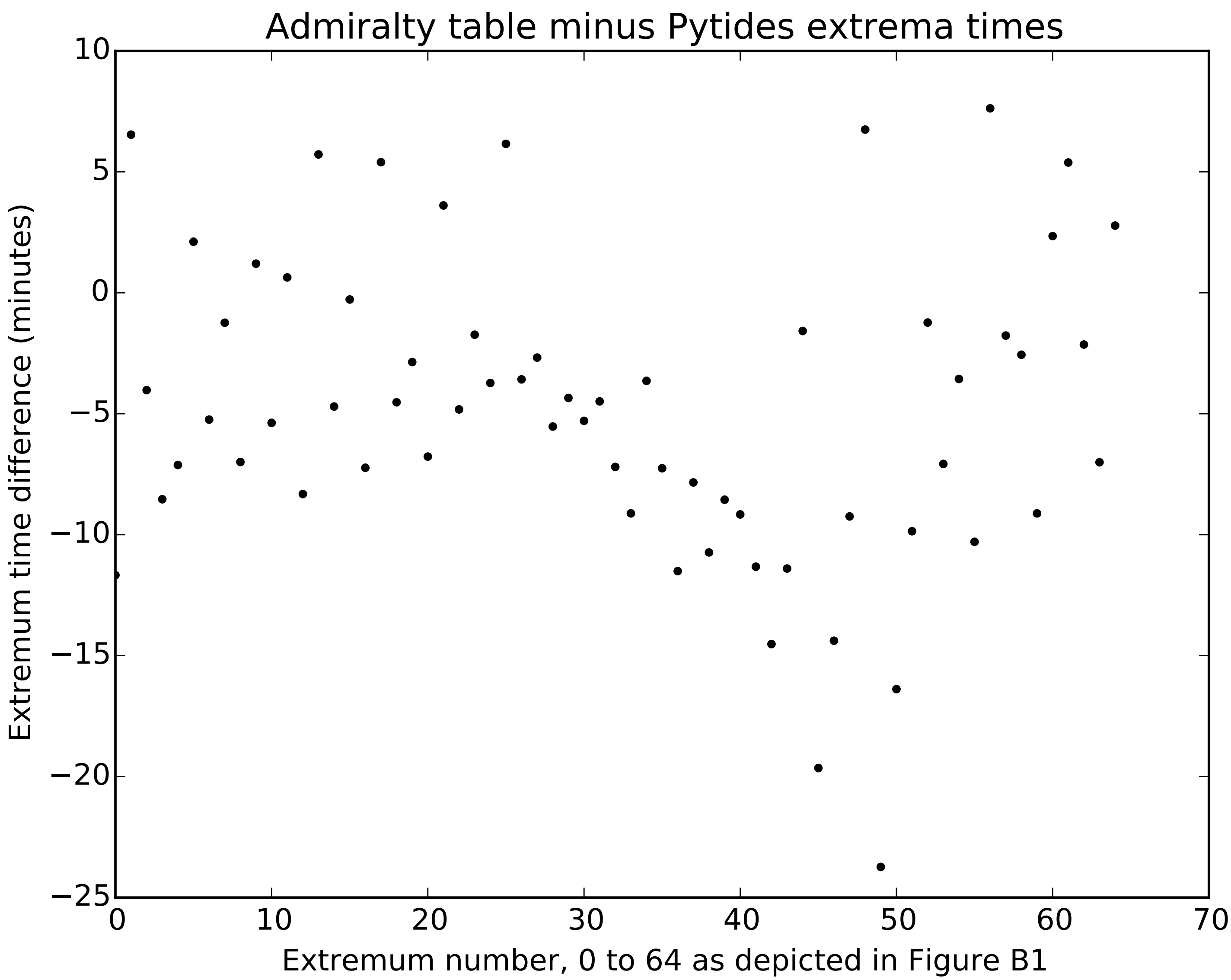
Pytides Tide using the International Hydrographic Bureau constituents (1966) from 320 hours pre-quake to 90 hours post-quake. The green circles are the Hydrographic Department (1944) published Admiralty table highs and lows.

Figure B4



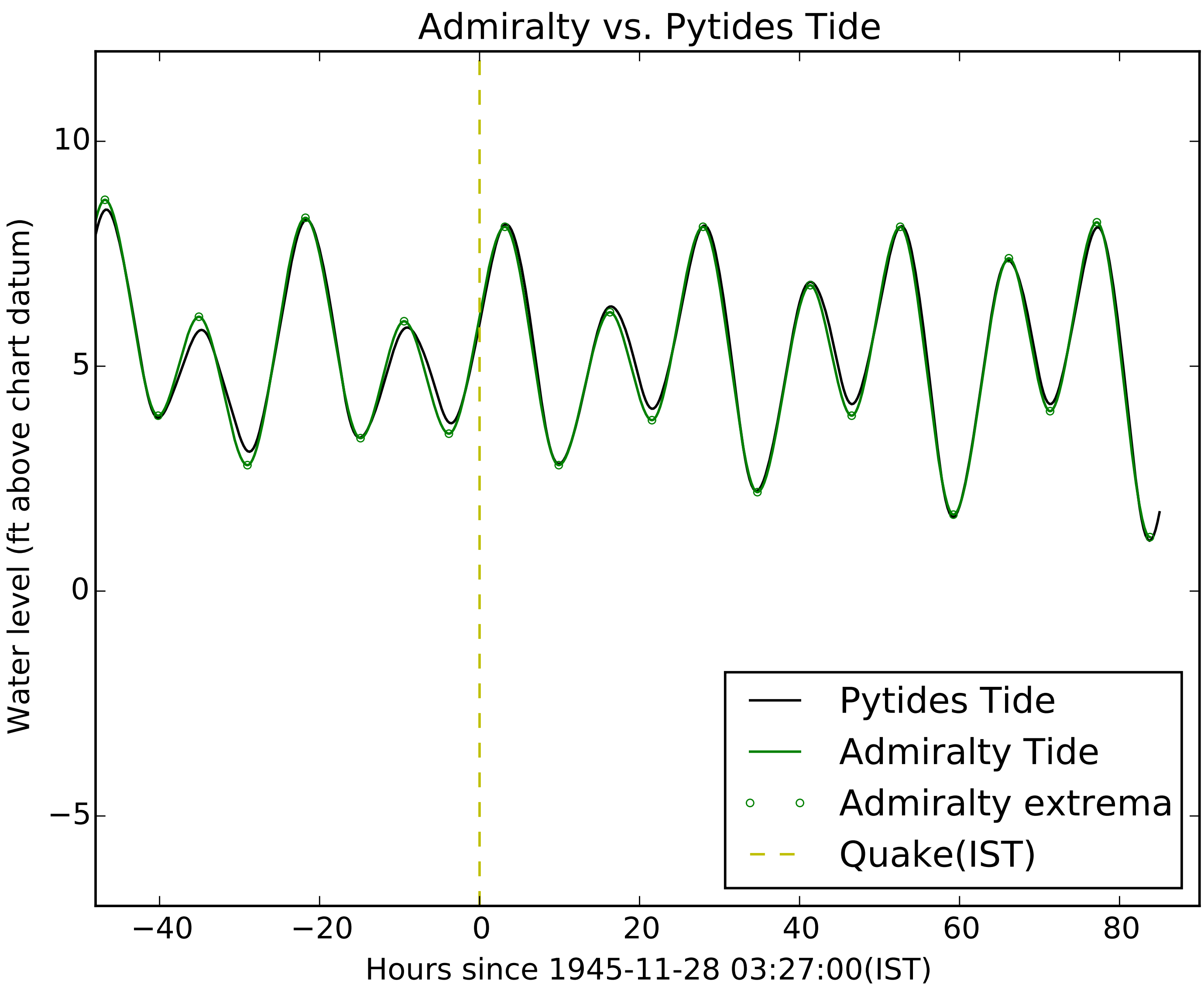
The 1944 Admiralty table minus the Pytides extrema heights. Pytides used the International Hydrographic Bureau (1966) constituents for Karachi. The difference is in feet, and the horizontal axis is the extremum number 0 to 64.

Figure B5



The 1944 Admiralty table minus the Pytides extrema times. Pytides used the International Hydrographic Bureau (1966) constituents for Karachi. The difference is in minutes, and the horizontal axis is the extremum number 0 to 64.

Figure B6



The Admiralty Tide created by connecting the 1944 Admiralty highs and lows with a piecewise cubic and the Pytides Tide created using the International Hydrographic Bureau (1966) constituents for Karachi from 48 hours pre-quake to 90 hours post-quake. The mean tide level was 5.21 feet.