The balance of autonomous and centralized control in scheduling problems

Supplementary Information

Henning Blunck[1*], Dieter Armbruster[2**], Julia Bendul[3***], and Marc-Thorsten Hütt[4*]

[*] *Jacobs University Bremen, Department of Life Sciences and Chemistry*
[**] *Arizona State University, School of Mathematical and Statistical Sciences*
[***] *RWTH Aachen University, School of Management & Economics*

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[1] h.blunck@jacobs-university.de
[2] armbruster@asu.edu
[3] bendul@scm.rwth-aachen.de
[4] m.huett@jacobs-university.de

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1 GCD Model Setup & Results

1.1 The Applied Network Dynamic

We apply here the the "AW" dynamic as defined by Hadzhiev et al. [1], which we will briefly summarize here:

A dynamic in Graph Coloring Dynamics is defined as the combination of a "temporal organization strategy" and a "neighborhood assessment strategy" [1] , where the first defines the sequence in which nodes can change their color over time and the latter describes the local decision heuristic applied in doing so.

As for the neighborhood assessment strategy, we apply a "strategic waiting" approach, which (given current color $c_i(t) \in \Sigma_C$ and color distribution in the 1-neigborhood $c_{N[i]}(t) = \{c_j(t)|j \in N[i]\}$) first generates a list of conflict minimizing colors. If $c_i(t)$ is in this list, $c_i(t+1) = c_i(t)$ with probability p ($p = 0.9$ for all experiments reported here), otherwise a color is chosen at random either from all possible colors (if $c_i(t)$ was the only conflict minimizing color) or from the set of conflict minimizing colors.

The experiments in [1] clearly indicate that only strategies that apply such a "strategic waiting" component can efficiently cope with networks of increasing complexity.

As for the "temporal organization strategy", both the results in [1] and our initial experiments confirm comparable performance between two possible attention propagation schemes: One that forwards attention to all neighbors of a node after a color change (A) and one that forwards the attention only to neighbors with which the new color establishes a conflict (C). We use the undirected attention propagation scheme A.

Both propagation schemes implement a three state approach at every node: Any node that is currently "quiescent", can be excited. This can be achieved either through excitation from a neighboring node or through spontaneous excitation[1]. In both cases, the node will change towards the "excited" state and execute its neighborhood assessment strategy in the next round. If this leads to a change in the node color, the excitement is passed on to all neighbors. After completing its neighborhood assessment strategy, the node changes to the "refractory" state where it stays for two rounds, until returning to the quiescent state. Attention propagation is hence a cue for the color reassessment (and potentially change) in the network.

1.2 The Transition between Connected and Unconnected Leader Nodes

In the main text we opted for the 3D representation of the performance as a function of problem complexity (i.e., chromatic number) and amount of centralized control (i.e., leader-to-ring connectivity), as the original hypothesis of the relationship between performance, level of autonomous control and task complexity was presented in this form in [2]. In Figures 1 and 2 we add the 2D version of Main Figure 2a, as well as the corrresponding figure for connected leader nodes.

[1] The probability for spontaneous excitation is set to $f = 0.05$ throughout our experiments.

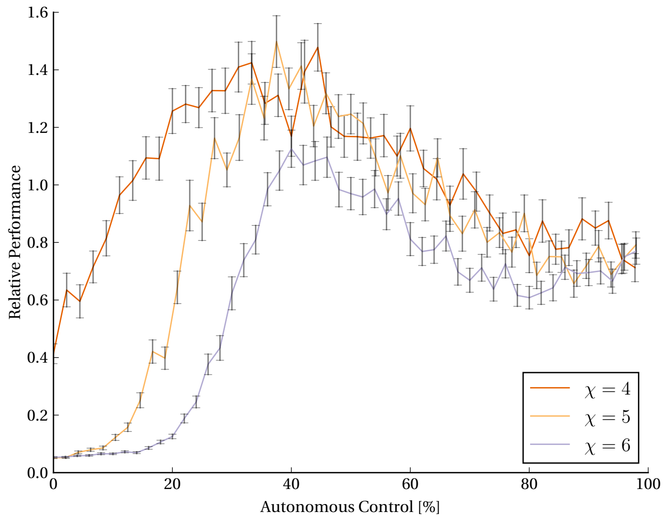


Figure 1: Performance as a function of the level of autonomous control for different chromatic numbers χ for unconnected leader nodes (data are the same as in Figure 2a of the main text).

To better show the impact of connected leader nodes on the relative performance attained in the GCD model, we present here a more detailed experiment: Instead of forming all possible edges between leader nodes or none at all, we increase the share of edges formed between leader nodes gradually, obtaining the result in Fig. 3.

The difference in behavior between connected and unconnected leader nodes is in fact systemic and can be observed over the entire range between completely unconnected and fully connected leader nodes.

The effect is also sustained across all investigated chromatic numbers. Fig. 4(a) is identical to Fig. 2(a) of the paper and shows the relative performance for unconnected leader nodes for the investigated range of chromatic numbers. Fig. 4(b) shows the same experiment but with fully connected leader nodes. Over all investigated chromatic numbers, the peak performance shifts significantly towards less autonomous control, when the leader nodes are connected.

2 Efforts to Understand the Model Behavior

2.1 Connected Leader Nodes Do Not Affect the Number of Solution Regimes

We already discussed in the paper that leader nodes do not "settle first". Their color change frequency does not decrease sooner as compared to ring nodes. Here, we show that their

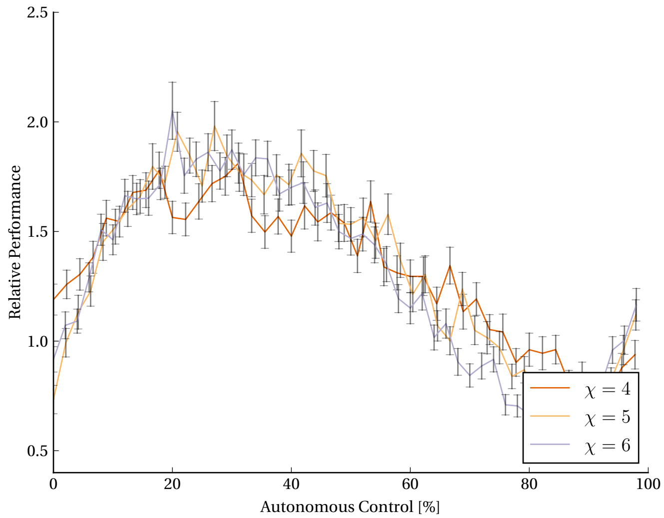


Figure 2: Performance as a function of the level of autonomous control for different chromatic numbers χ for connected leader nodes (data are partly shown in Figure 2b of the main text).

involvement in the solution process also does not necessarily decrease the average number of solution regimes on the ring (Fig. 5).

This is further evidence that the coordinating effect of leader nodes is largely in the improved removal of remaining conflicts, once an initial distribution of solution regimes has spread large parts of the ring.

2.2 A Closer Look at Information Transfer

In all experiments on information transfer, we calculate the mutual information between the discrete time series given by the leader node state at round t as compared to the state of a connected ring node at round $t+1$.

In the publication, we measure the mutual information, using a binary alphabet (whether or not a node is in a solution regime). . Here, we also present the results when performing the analysis based on node color and discuss how solution regimes can be identified for leader nodes (c.f. Sec. 2.2.2).

2.2.1 Alternative Measure: Information Transfer Based on Node Color

The obvious first alphabet on which information transfer should be investigated is that of the node colors. However, as Fig. 6 demonstrates, no striking effect explaining in particular

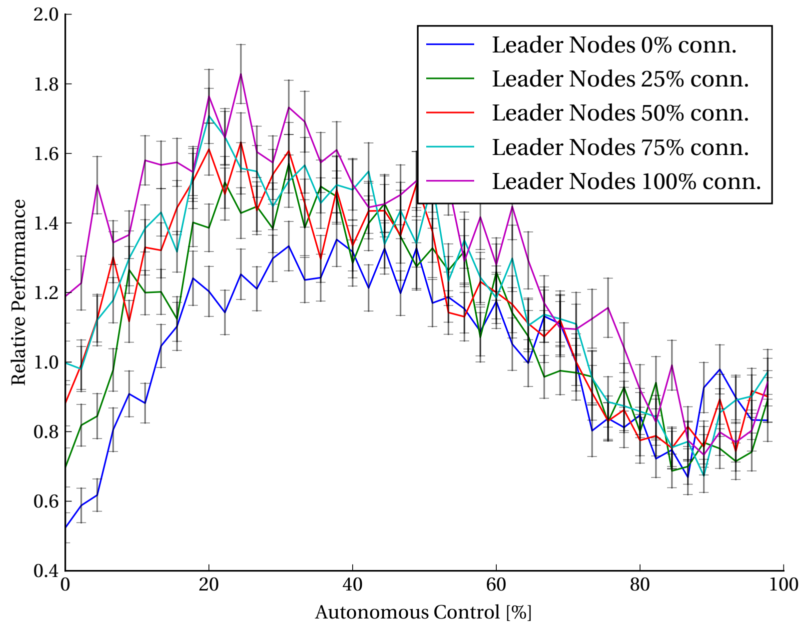
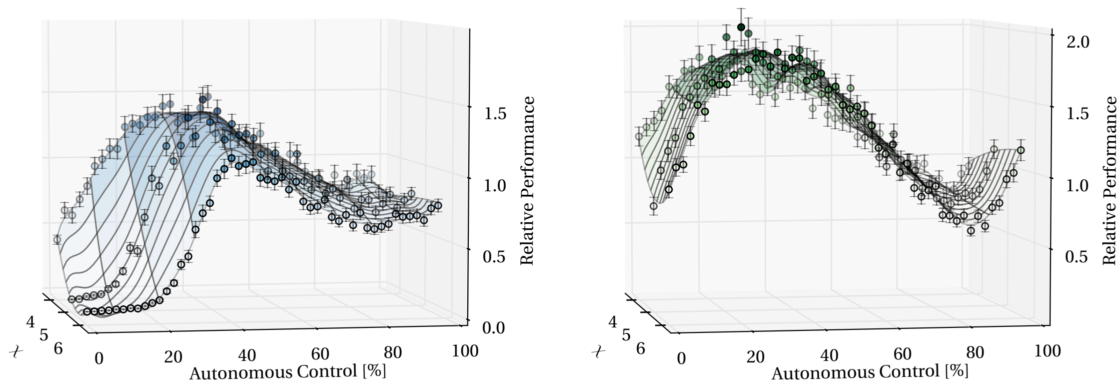


Figure 3: Relative performance as a function of leader node connectedness (with ring nodes) for different levels of connectedness amongst leader nodes.



(a) (b)

Figure 4: Relative performance over investigated range of chromatic numbers for unconnected leader nodes (a) and fully connected leader nodes (b).

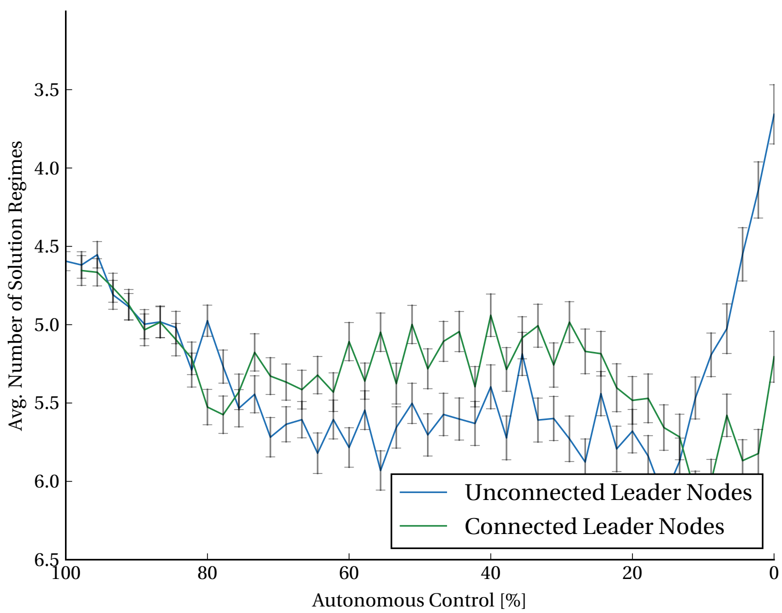


Figure 5: Average number of solution regimes on the ring in the mid 20% of the solution process. y-Axis is inverted, since fewer solution regimes are closer to a feasible solution and hence considered "better".

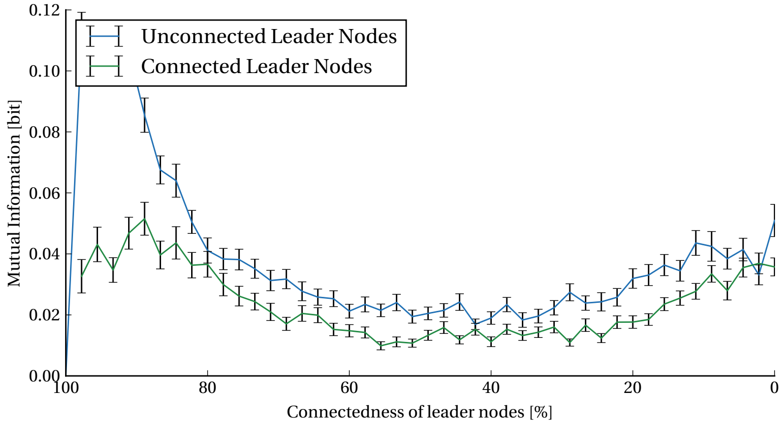


Figure 6: Average mutual information flow between leader node and connected ring nodes, based on node color.

the better performance of networks with interconnected leader nodes can be observed.

This result is further evidence that communication between leader nodes improves the formation of regimes rather than their interoperability along the ring.

2.2.2 Assigning Solution Regimes To Leader Nodes

As defined in the methods section, a (ring-) node is considered to belong to a solution regime, *if and only if* the node itself and the $\chi - 1$ nodes following (in the ring) have different colors (and hence cover the entire solution space Σ_C). Otherwise, the node is believed to belong to no solution regime.

Likewise, a leader-node is believed to belong a solution regime, if the χ leader nodes have different colors. Hence, all leader nodes can either be part of one single solution regime collectively (if they have χ different colors) or none at all. Note that the colorability preserving installation of leader node links also gives the leader nodes an inherent order and hence makes it possible to measure assign them a solution regime as well.

2.3 Noise Emitting Leader Nodes

2.3.1 Estimating the Excitement Probability

We measure the share of leader nodes being "excited" each round and the number of conflicts during that round. The resulting averages over multiple runs of the same setup are plotted in Fig. 7 for both unconnected and connected leader nodes. A third degree polynomial function is fitted to the data.

The rather poor fit for high numbers of conflicts is negligible, since, as shown in Fig. 3(a) of the paper, the conflict count decreases rapidly during the first, transient phase of the

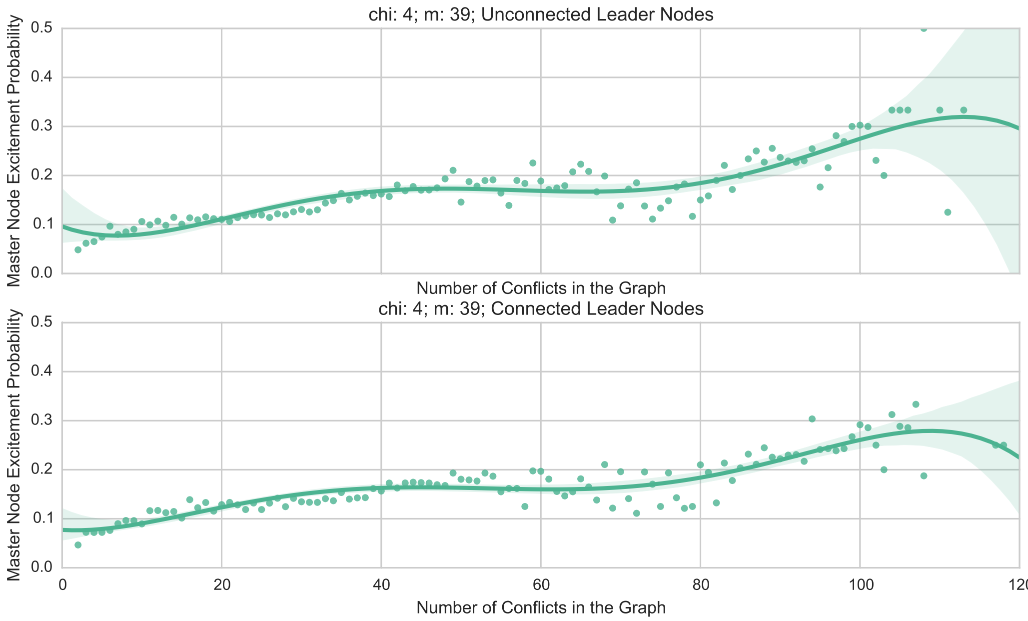


Figure 7: Leader node excitation probability for networks with $\chi = 4, N = 60, s = 30, m = 39$ with polynomial of degree 3 fitted.

solution process, and hence the excitement probability estimate for high conflict numbers is (a) based on few observations and (b) of little importance for the solution process since the leader nodes play little role in the first, rapid reduction of conflicts.

2.3.2 Excitation Probability for Unconnected Leader Nodes

For unconnected leader nodes, each leader node is externally excited in a given round with probability $p = f(c(t))^{\frac{1}{\chi}}$ where $f(x)$ is the fitted function, evaluated at the current number of conflicts $c(t)$ and χ is the chromatic number and hence the number of leader nodes.

References

[1] B. Hadzhiev et al. "A Model of Graph Coloring Dynamics with Attention Waves and Strategic Waiting". In: *Advances in Complex Systems* 12.6 (2009), pp. 549–564. DOI: 10.1142/S0219525909002386.

[2] Michael Hülsmann, B Scholz-Reiter, and Katja Windt. *Autonomous cooperation and control in logistics.* Heidelberg, Germany: Springer, 2011. ISBN: 9783642194689.